THE PROBLEM OF INDIVIDUAL DIFFERENCES IN THE TEACHING OF SECONDARY-SCHOOL MATHEMATICS

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In making a study of practice teaching in secondary mathematics the writer sent a questionnaire to the various institutions which are at present offering practice-teaching courses. The general aim of the survey is to reveal and organize the technique involved in the administering of a course in practice teaching of high-school mathematics.

The question which was answered with greatest interest deals with the general problem of individual differences. In the survey the question was stated as follows: "What instructions are given (to practice teachers) to meet individual differences of high-school students?" In order to focus particular attention to the two extremes the two following duplicating questions were included: "(1) How is the practice teacher taught to deal with the slow student? (2) With the fast worker?"

The writer proposes (1) to present a list of current practices as revealed by the responses to the questionnaire, (2) to study the validity of these practices, (3) to discuss in detail the proper technique involved in dealing with the slow student. Practical devices aiming to meet the needs of the fast worker will be presented in a later number of the *School Review*.

The reader would find it most interesting and profitable to read the forty-five replies received. Obviously space is here not available. It is possible here to submit a few typical examples, to present a summary of current practices gleaned from this material, and to give the assurance that in the near future this material will be published in another connection.

TYPICAL RESPONSES

INDIANA STATE NORMAL

a) Careful distribution of questions; suggestions for additional work by strong pupils; occasional permission for strong pupils to assist weaker pupils; emphasis on the fact that pupils are to be held for individual and not class standards of work.

b) To give especial attention to the slow pupil when a new topic is being developed; to give him individual help in class and occasional help outside of recitation period. The presence of weak pupils in every class necessitates a careful and slow approach to each new topic.

TEACHERS COLLEGE, COLUMBIA

To direct the teaching of new matter especially to him (the slow student). To drill to make sure the slow student understands. Use of bright students to help slow students.

UNIVERSITY OF SOUTHERN CALIFORNIA

Possibilities of the course are always higher than the requirements. In this way the better students are taken care of. Personal help after hours is given to slower students.

COLORADO COLLEGE

Very often slower students are assigned to the practice teacher individually.

UNIVERSITY OF NORTH DAKOTA

Keep bright pupils busy with extra work. Do not hold back the class for the slowest pupil nor go as fast as the brightest one is able to go.

LAKE ERIE COLLEGE, OHIO

The poorer ones are called on for additional oral drill while others are doing board work which is later to be explained to the whole class.

UNIVERSITY OF TEXAS

The teacher sets a minimum for the slow student and encourages each student to do as much as possible beyond this.

UNIVERSITY OF OKLAHOMA

[Reply similar to that of University of Texas.]

PARK COLLEGE, MISSOURI

If possible ascertain the cause. If he is asleep, wake him up. If he is discouraged, coax him. If he is stubborn, rebuke him. If he is physically unable, be very gentle and considerate. [No suggestion is given as to the procedure when student is mentally unable to do the work.]

UNIVERSITY HIGH SCHOOL, CHICAGO

A supervised study class of slow students running throughout the year as a part of the regular program, interscholastic contests, mathematics exhibits, and special work in writing themes dealing with topics of interest to the particular student.

The various methods of dealing with the slow student are: (1) conferences with practice teachers in hours other than class periods. It is obvious that here practice teachers can be of valuable assistance in return for the extra work which they themselves cause the regular instructor. However, this must be definitely organized or the result will be overloading a practice teacher with unprofitable labor. (2) Keeping in mind a definitely organized minimum course of study accompanied by an attempt to have every student accomplish something beyond this. This involves an analysis of the student's difficulty, and the grading of the work commensurate with ability of individual members of class. (3) Supervised classes of study for slow students. (4) Use of the bright students as instructors of the slow students. (5) Division of class on the basis of ability into separate sections. (6) Supplementary problems assigned to fast workers.

MINIMUM COURSES

It is interesting to note that the replies to the survey reflect the importance that is being attached to minimum courses for the different years of high-school mathematics. In recent years the demand for such a course has been particularly insistent. This demand is intimately connected with the subject of "standard units." The advantages of a definitely organized minimum course are: (1) It enables the teachers to adjust work to meet the needs of individual students. (2) It puts in definite form before the student the least that must be accomplished in order to receive a mere passing grade. (3) It forces the attention of teacher and pupils upon the essentials of the course. (4) A minimum course conveys a definite interpretation of the phrase "a year of mathematics" to secondary-school executives and college-admission boards. An examination of current textbooks of secondary-school mathematics leads to the conclusion that it would be easily possible for a national committee to come to definite

agreements as to the knowledge of certain essential topics which should constitute a standard minimum course. Definite suggestions as a basis for such actions are found in the "Tentative Proposals for Minimum Courses," published in *A Review of High-School Mathematics* by Reeve and Schorling (University of Chicago Press). These minimum courses are organized on the basis of a study of textbooks in extensive use.

In the University High School, Chicago, the minimum course bears an intimate relation to the supervised classes of the slow students. All students "below passing" are assigned to a special class of supervised study. "Below passing" is defined as failure to meet the requirements of a definitely organized minimum course for the particular grade.[1] The student reports for such periods until he meets the minimum requirements. If he succeeds in merely meeting the requirements of the minimum course for the year he is given a credit with the sentence attached, "Student must repeat course if he wishes to continue work."

This credit permits student to graduate from high school, but brings pressure to bear upon him to find sòme subject that he can do with greater profit and success than he has been able to do the course in mathematics with two hours of supervised study throughout the year. On the face of it the scheme would appear to have the effect of reducing the election of mathematics. Statistically this result is not apparent after three years of operation. It is possible that the formulation of a definite minimum goal has the opposite effect.

SECTIONS BASED ON ABILITY

The division of the class into sections on the basis of ability is a method of solving the problem of individual differences that is promising for future experimentation. One argument in its favor is that it takes care equally well of the bright student and the slow student. It is certainly true that the bright student is entitled to as careful consideration as the slow student. It is possible that the enthusiasm of many brilliant first-year mathe-

[1] For a detailed description of the method of conducting special study classes see Breslich, "Supervised Study," *Thirteenth Yearbook of the Society for the Study of Education.*

matics students is killed by neglect. It is to be regretted that the institutions reporting that they employ the method of dividing the classes on the basis of ability did not report results in greater detail. In every case the experiment is described in general terms.

In a number of schools in the Middle West, e.g., Cicero Township High School, Illinois, and Elkart, Indiana, the method is employed and the claim is made that it operates to the benefit of both the fast workers and the slow workers. If this is true, then the reorganization of mathematics classes of large school systems on this basis should certainly not be delayed. It is to be hoped that definite results may be published by the schools which are operating under this system.

BRIGHT STUDENTS AS INSTRUCTORS OF SLOW STUDENTS

The use of bright students as instructors of the slow workers raises certain fundamental questions: (1) Is there tangible evidence that the slow student is really benefited? (2) Is the process to be learned shown by the bright student as an abstract process or is it taught to the slow student in a rationalized form? (3) Just what effect has classroom communication on mathematical subjects by the members of the class on the learning process? The writer believes that these questions should be answered by definite experiments.

The writer carried out an experiment in the year 1913–14 with the hope of having these questions answered in his own mind. Two classes in the second semester of second-year mathematics (geometry emphasized in this course) were used in the experiment. The two classes recited in successive periods. The following groups were arbitrarily ruled out of the experiment: (1) students who were repeating the course, (2) students who were tutored or assigned to supervised-study classes, (3) students who dropped the course before the close of the experiment or who were absent five or more days during the experiment. This elimination left twenty students in each section.

The relative ability of the two classes and the ranking of the individual students were determined. Since the instructors in the University High School report grades of pupils in multiples of

5, the students of the two classes were distributed in various groups corresponding to the multiples of 5. The group to which an individual pupil was assigned was determined by adding the grades of the three preceding semesters of high-school work and the mid-semester grade of the current semester. This sum was divided by 4 and the student assigned to the multiple of 5 group nearest this quotient; e.g., a student having the grades 95, 95, 90, 95, with a sum 375 and a quotient of 93.75, was assigned to 95 or A+ group. Table I shows the relative standing of the individual students and the comparative strength of the two classes. The first class will be termed the A section and the class reciting the next period the B section.

If the number of students in each group is multiplied by the number at the top of each column and if these products are added, Section A totals 1,580 points whereas Section B totals 1,540. The inference of these figures is that Section A was the stronger section by 40 points. The table also shows a more nearly uniform distribution for Section A. These conclusions agreed with the general opinion held by the mathematics faculty concerning these two sections.

A series of twelve lessons, identical in content, was given to the two sections. The instructor taught each lesson inductively at the beginning of each period. This occupied approximately one-third of the recitation period. For the remaining two-thirds of the period the pupils of Section B worked independently under the supervision of the instructor. Most of the work was done in notebooks at desks. The pupils were also requested to do all home work independently. It is believed that this request was carried out to the desired degree. On the other hand the students of Section A were asked to work in groups at desks and at blackboard. Here advantage was taken of natural groupings. In such cases where pupils did not voluntarily work in groups, they were arbitrarily paired at desks. Furthermore, students were asked to gain as much help as possible from each other in the preparation of home work. In short, the instructor discouraged independent work in the last thirty minutes of each recitation and in the preparation of home work. After eight recitations the two classes

TABLE I

	60	65	70	75	80	85	90	95
Section A	Van	Agar Rogers	Carry Smith Falkenau	Albright Reber Antoine	Frazier Church Stieglitz	Kimball Howe Noble	Friedman Mallory Zeisler	Edwards Schulman
Section B	Bolte Miller	Sprochule Strauss Sully	Berg Porter	Bensley Eisenrath Hummel	Sulzberger Budinger Foreman	Van Pelt O'Connor Kuh	Clark Mayer Davis	Fake

were given the same examination covering two successive periods. A second identical test was given to both classes at the end of twelve recitations. All papers were graded by colleagues of the instructor. The students were distributed in groups by the same method which determined the relative strength of the two classes. Table II reflects the result of these two examinations.

STUDY OF TABLE II

1. Multiplying the number of students in each group by the figure at the top of each column shows Section A to have a strength of 1,540; Section B, 1,515.

2. Section A has a total negative displacement of 16, a positive displacement of 8, total 24. Section B has a negative displacement of 9, a positive displacement of 4, total 13. The positive displacements of Section A are not numerous in the upper ranks; those of Section B reach as far down as 70.

3. Section A no longer shows a uniform distribution; the 60, 65, and 70 groups are almost depleted. Section B shows approximately the same degree of uniformity as in Table I.

The most obvious fact is that something has been functioning as a disturbing factor in the uniform distribution of Section A. This disturbing influence has been operating negatively among the students at the two extremes, the slow workers and the very good students. A few students in ranks 65 and 70 have been helped but the greater number have been crowded into lower ranks. A considerable number of the good students have been helped, but some of the very best members of the class have for the first time in four semesters of high-school work been unable to hold their positions. Finally, it is probably true that this disturbing factor is the change in the method of teaching Section A.

At the end of the series of lessons described above the method was reversed; i.e., after the new material had been taught by identically the same method to both classes, Section A now worked independently and Section B worked in groups. The results of examinations given at the close of the experiment are shown in Table III.

TABLE II

	Below 60	60	65	70	75	80	85	90	95
Section A working in groups	Van (−3)	Agar (−1) Rogers (−1) Smith (−2) Falkenau (−2)	Antoine (−2)		Frazier (−1)	Carry (2) Albright (1) Reber (1) Kimball (−1)	Church (1) Friedman (−1) Mallory (−1) Stieglitz (1)	Noble (1) Howe (1) Edwards (−1) Zeisler	Schulman
Section B working independently	Miller (−2) Bolte (−2)	Hummel (−3)	Strauss Sully	Bensley (−1) Sprochule (1) Berg	Porter (1) Eisenrath	Budinger Foreman Sulzberger	Kuh Mayer (−1) Van Pelt	Clark O'Connor (1)	Fake Davis (1)

The figures in parentheses indicate the number of displacements from the position held in Table I; e.g., (−2) after Smith of Section A indicates that the grade received is 10 per cent lower than the office records show in Table I, and (1) after Porter of Section B indicates that Porter now in the 75 group is 5 per cent higher than his position in Table I. The (−2) after Bolte indicates an average of 50 on the three-day examinations.

TABLE III

	Below	60	65	70	75	80	85	90	95
Section A working independently	(50) Van (−2)	Agar (−1)	Smith (−1) Antoine (−2)	Rogers (1) Falkenau Frazier (−2)	Church (−1) Albright Carry (1)	Reber (1) Zeisler (−2)	Stieglitz (1) Kimball	Noble (1) Mallory Friedman	Schulman Edwards Howe (2)
Section B working in groups	(55) Strauss (−2) (60) Hummel (−3) (50) Miller (−2) (50) Bolte (−2)	Sully (−1)	Bensley (−2)	Eisenrath (−1)	Porter (1) Sprochule (2)	Mayer (−2) Sulzberger Berg (2)	Foreman (1) Clark (−1) O'Connor Davis (−1)	Kuh (1) Budinger (2) Van Pelt (1)	Fake

1. Section A now shows a strength of 1,560 as compared with 1,505 for Section B. The sections are now farther apart than at any other time. It is true that the totals of the two tables are not comparable; however, in the last examination Section A was able to raise its total grade from 1,540 to 1,560, while Section B experienced a further drop from 1,515 to 1,505.

2. Section A is regaining its former uniformity, whereas Section B now shows all the evidences of the disturbing factor of a change of method revealed by Table III for Section A.

3. Section A shows a displacement of (−11) from the corresponding positions in Table I, a positive 7, total 18. This is a gain of 5 in the negative and a loss of 1, or an advance of 6 toward its former strength. On the other hand Section B has 17 negative displacements and 10 positive, total 27. The negative displacement is 8 units larger and the positive 6 units larger. Moreover the distribution of these displacements is the important characteristic. It is possible that this weaker section is more seriously affected by the change of method. Furthermore, Section A may still be laboring under the effects of the first part of the experiment. If so the results are all the more significant.

4. The following interesting facts were discovered: (1) Of the students in Section A in rank 75 or higher, 71 per cent preferred to work independently; in Section B 77 per cent of the same type preferred independent work. (2) Of the students in Section A of rank lower than 75, $66\frac{2}{3}$ per cent preferred group work, and the same was found to be true of 70 per cent of Section B. This was in spite of the fact that a large number received lower grades. This preference may be due to the fact that the group study enabled them to work with greater comfort in class inasmuch as it was possible for them to conceal their weakness in the work which was partly some other student's. (3) The students in groups 75 or higher were in no way influenced by the views of the instructor as to what constitutes a proper atmosphere for effective classroom work. The instructor preferred decidedly an informal recitation which barely falls short of confusion. (4) Short tests of two questions were given to each section ten recitation days after the same question had been given on the previous tests involved in the fore-

going tables. The purpose was to test the comparative amount of information retained by the two sections. The average of Section A (group work) was 68.2; of Section B (independent) 70.3. On the second test Section A (working independently) averaged 76.6 and Section B (group work) averaged 69.9. In both cases the short tests were given without review or warning. If a generalization is possible it must certainly be that mathematical material gained by independent study is retained to a higher degree. (5) The last part of the experiment was difficult to carry out because the material did not adapt itself readily to group work. The teaching could not possibly be done in one-third of the time in either section and in many cases the supervised study was a mere fraction of the total time. It is obvious that the method of the recitation is not independent of subject-matter; on the contrary, method is dictated by subject-matter.

The generalization of this experiment is not conclusive. Indeed, that is itself one of the generalizations. This discussion emphasizes the necessity of further experiments along the same lines on the part of the schools which report that they are using the fast workers as helpers of the slow workers as an effective means of attacking the problem of individual differences. It is just possible that the fast workers are not only not helping the slow workers but are actually weakening them. If so it means that the mathematical theory in question is shown as an abstract method of getting desired results and is not taught as a rationalized process. Furthermore it is possible that the fast worker is seriously retarded and that interest is decreased. It may be far more profitable for him to advance to new work or spend the time in applying theory to new problems than to be "showing" his classmates problems which he has thought through. At any rate the one point here emphasized is that the assumption that both the slow student and the fast worker are profited by the method is unwarranted.

FUNDAMENTAL ASPECTS OF THE PROBLEM

Thus far we have discussed four of the five current practices which affect the slow worker. We shall now turn to the discussion of the three fundamental principles which underlie these methods.

The principles upon which a satisfactory solution of the problem must be based may be formulated as follows:

(1) Regulating the rate of the presentation of new subject-matter to the ability of the majority of the class regardless of the rate at which those at the extremes, the unusually fast or very slow workers, are able to progress. This is to be accomplished by the consciousness of well-defined standards on the part of the teacher. (2) Providing extra instruction in supervised study for the slow workers. This instruction may be given either as part of the regular recitation, or in a special period which constitutes a part of the regular program; fundamentally this means teaching the slow worker how to study. (3) Providing profitable supplementary activities for the fast workers that will stimulate their enthusiasm for the subject.

Corresponding to the three principles set forth, there are three types of teachers: (1) The one who teaches to the greater number falling between the extremes, neglecting the failing students either because he does not have time to give them the required special attention or because he believes that a certain number are doomed to failure anyway or because it is necessary to fail a considerable number in order to justify the claim that he is giving a rigorous course, and neglecting the excellent students because he believes they will advance in spite of his neglect. (2) The conscientious plodder who believes it to be his special mission to teach the slow student. This is the type of teacher who will hold up the advance of a whole class by an inductive series of questions directed to the one student who fails to see the point. A common characteristic of this teacher is his inability to keep up with the schedule of the department, under the plea of thoroughness. (3) The teacher who directs his lessons directly to the excellent student. He loves his subject and moves at a constant, accelerated rate because of the reaction he gets from the few fast workers. At the end of the semester he either fails an indefensibly large number or passes a great number who cannot carry the work with the next instructor. The instructor of university mathematics needs to guard against the tendency to fall into this group.

TEACHING THE SLOW WORKER HOW TO STUDY

Special reference has been made to the complete survey and experiments of supervised study of slow workers made by Breslich. Fundamentally the problem consists in teaching pupils how to study. The difficulty of the problem and the psychological principle that underlie the solution are set forth in detail in Judd, *The Psychology of High-School Subjects*. Parker, in *Methods of Teaching in High Schools*, formulates practical pedagogical principles of supervised study with numerous examples of applications of these principles. The writer believes it worth while to direct attention to a few of the more significant attempts to solve this problem on the practical side.

Perhaps the best known of these studies is the Batavia plan, organized by Superintendent J. Kennedy of Batavia, New York. This scheme places a number of required supervised-study periods in the daily program and requires the teacher to use them for directing pupils who study silently at desks. A modified form of this plan is found in Joliet, Illinois, where the regular forty-minute period set aside for instruction in mathematics is followed by a second period devoted to supervised study. In a number of places where the Batavia plan or modified forms have been used, teachers insist on using the whole time given them for recitation purposes of the traditional type unless the supervised study is made a requirement. This tendency on the part of teachers can be overcome if teachers in training are taught to distribute the supervised study throughout the recitation period, giving such work at those psychological moments when it will function most effectively.

A third method is illustrated by the Reavis experiment. In this experiment each student makes out a definite program of study for the term. The school brings such pressure to bear upon the student as will make the student conscientiously attempt to carry out this program. The measured evidence submitted by Mr. Reavis is worthy of careful study.

A formal attempt to teach pupils how to study is illustrated by the "Study Helps" formulated by the faculty of the University

High School, Chicago. A copy of "Study Helps" is submitted below. These definite suggestions are for the most part due directly to the experience of supervising teachers in supervised-study classes of slow workers in mathematics. Valuable suggestions were given by the school librarian. A few are the suggestions of professors of psychology and methods in the School of Education.

STUDY HELPS

For Students in the University High School

The habits of study formed in school are of greater importance than the subjects mastered. The following suggestions, if carefully followed, will help you make your mind an efficient tool. Your daily aim should be to learn your lesson in less time, or to learn it better in the same time.

1. Make out a definite daily program, arranging for a definite time for each study. You will thus form the habit of concentrating your thoughts on the subject at that time.

2. Provide yourself with the material the lesson requires; have on hand maps, ruler, compass, special paper needed, etc.

3. Understand the lesson assignment. Learn to take notes on the suggestions given by the teacher when the lesson is assigned. Take down accurately any references given by the teacher. Should a reference be of special importance, star it so that you may readily find it. Pick out the important topics of the lesson before beginning your study.

4. In the proper use of a textbook, the following devices will be found helpful: index, appendix, footnotes, maps, illustrations, vocabulary, etc. Learn to use your textbook, as it will help you to use other books. Therefore understand the purpose of the devices named above and use them freely.

5. Do not lose time getting ready for study. Sit down and begin to work at once. Concentrate on your work, i.e., put your mind on it and let nothing disturb you. Have the will to learn.

6. In many kinds of work it is best to go over the lesson quickly, then to go over it again carefully; e.g., before beginning to solve a problem in mathematics read it through and be sure you understand what is to be proved before beginning its solution; in translating a foreign language, read the passage through and see how much you can understand before consulting the vocabulary.

7. Do individual study. Learn to form your own judgments, to work your own problems. Individual study is honest study.

8. Try to put the facts you are learning into practical use if possible. Apply them to present-day conditions. Illustrate them in terms familiar to you.

9. Take an interest in the subjects taught in school. Read the periodical literature concerning these. Talk to your parents about your school work. Discuss with them points that interest you.

10. Review your lessons frequently. If there were points you did not understand, the review will help you to master them.

11. Prepare each lesson every day. The habit of meeting each requirement punctually is of extreme importance.

METHOD OF IMPRESSING STUDENTS WITH THE IMPORTANCE OF "STUDY HELPS"

When the faculty had adopted the "Study Helps" a special day was set for a campaign designed to impress the student with the importance of these suggestions. Each instructor distributed copies to the students and requested that they be pasted in the textbooks. Copies were posted on bulletin boards. A part of every recitation throughout the day was devoted to the discussion of the eleven suggestions. Special attention was given to them in the library. In later recitations a number of the faculty led informal discussions calculated to discover the value received by the members of the class. The English department used "Study Helps" as a subject for a theme.

The scheme was considered seriously by the great majority of students, particularly the upper classmen. It is not possible to submit measured results showing the efficiency of this scheme. However, it is the consensus of opinion among faculty members and parents that much good was derived.

Mathematics teachers realize the importance of the problem of individual differences. The programs of recent meetings are sufficient evidence. In accord with our American ideals most of the efforts have been directed to the needs of the slow worker. This article aims to supplement the valuable articles on that phase of the subject which have recently been published. On the other hand, the fast worker has been wholly neglected in spite of the general conviction that he is entitled to consideration. In a later article the writer proposes to discuss practical devices aiming to meet the needs of the fast worker in mathematics.